Designing and Evaluating an Interpretable Predictive Modeling Technique for Business Processes

Dominic Breuker[1], Patrick Delfmann[1], Martin Matzner[1] and Jörg Becker[1]

[1] Department for Information Systems, Leonardo-Campus 3, 48149 Muenster, Germany
{breuker, delfmann, matzner, becker}@ercis.com

Abstract. Process mining is a field traditionally concerned with retrospective analysis of event logs, yet interest in applying it online to running process instances is increasing. In this paper, we design a predictive modeling technique that can be used to quantify probabilities of how a running process instance will behave based on the events that have been observed so far. To this end, we study the field of grammatical inference and identify suitable probabilistic modeling techniques for event log data. After tailoring one of these techniques to the domain of business process management, we derive a learning algorithm. By combining our predictive model with an established process discovery technique, we are able to visualize the significant parts of predictive models in form of Petri nets. A preliminary evaluation demonstrates the effectiveness of our approach.

Keywords: Data mining, process mining, grammatical inference, predictive modeling

1 Motivation

In recent years, business analytics has emerged as one of the hottest topics in both research and practice. With stories praising the opportunities of data analysis being omnipresent, it is not surprising that scholars from diverse fields explore how data can be exploited to improve business understanding and decision making. In business process management (BPM), a research discipline called *process mining* has emerged that deals with analyzing historical data about instances of business processes [1]. The data usually comes in form of event logs, which are collections of sequences of events that have been collected while executing a business process. Events are of different types. The type describes the nature of an event. For instance, an event could represent an activity that was performed or a decision that was made.

Traditionally, process mining has been used mainly for retrospective analysis. The primary task of many process mining techniques is to discover the underlying process given an event log, with the goal of providing analysts with an objective view on an organization's collective behavior [2]. Further analyses can follow. One could verify that a process conforms to a given specification or improve the process based on the discovered insights [1].

In recent times, interest has been expressed into applying process mining no only ex post but also in an online setting. The idea is to apply insights gained from event logs to currently running process instances. The goals could be to predict properties of interest or recommend good decisions [3]. Techniques have been designed to predict the time at which a process instance will be completed [4] or to recommend performers for tasks such that performance is maximized [5].

In this paper, we consider a similar prediction problem. Interpreting events of an event log as outcomes of decisions, our goal is to build a predictive analytics model that accounts for the sequential nature of decision making in business processes. Provided with a partial sequence of events from an unfinished process instance, the model's task is quantifying the likelihood of the instance ending with certain future sequences. The model should be trained solemnly by means of event log data.

Such a predictive model could be useful in variety of situations. For instance, early warning systems could be built. Decision makers could be provided with a list of running process instances in which undesirable events will likely be observed in the future. A predictive model could also be used to estimate how many running instances will require a certain resource, again to warn decision makers if capacities will be exceeded. Moreover, anomaly detection approaches could be built to identify highly unlikely process instances. They could help pointing business analysts to unusual instances. It is also conceivable to use a predictive model to support other process mining endeavors, for instance by imputing missing values in incomplete event logs.

The first step in constructing a predictive modeling approach is defining a suitable representation of the data and the model structure. To find one, we tap into research from the field of grammatical inference, which is the study of learning structure from discrete sequential data [6], often by fitting probabilistic models [7]. After identifying a suitable model, we also discuss how to fit the model to data. Again, we draw on grammatical inference to identify a suitable technique. Subsequently, we adapt the probabilistic model to the BPM domain. As a consequence, we have to adapt the fitting technique too. By applying the predictive modeling approach to real-world event logs, we illustrate how it can be used and demonstrate its effectiveness.

The primary goal of our modifications is to define the probabilistic model such that its structure most closely resembles the structure of business processes. An advantage of that is that models with appropriate structure typically perform better when used in predictive analytics. The other goal is to keep the model interpretable. Whenever non-technical stakeholders are involved in predictive analytics endeavors, interpretability can be crucial to get them on board. For this reason, interpretable models may be preferred even if they perform worse than their black box counterparts [8]. By combining a technique from process mining [9] with our predictive modeling approach, we are able to visualize a predictive model's structure in form of process models. This puts domain experts in the position to judge the quality of a predictive model.

The remainder of this paper is structured as follows. In section 2, we discuss probabilistic models and fitting techniques used in grammatical inference. Section 3 is devoted to the BPM-specific modifications we designed. In section 4, we describe how process mining can be applied to create visualizations. The evaluation of our approach is documented in section 5. In section 6, we conclude and give an outlook to future research.

2 Grammatical inference

2.1 Probabilistic models of discrete sequential data

The inductive learning problem tackled in grammatical inference [10] starts with a data set $X = \{x^{(1)}, \dots, x^{(C)}\}$ with C strings. Each string $x^{(c)}$ is a sequence $\left(x_1^{(c)}, \dots, x_{t_c}^{(c)}\right)$ of symbols t_c, all of which are part of an alphabet: $x_t \in \{1, \dots, E\}$. The number of different symbols shall be denoted E. When interpreting the data as an event log with C process instances, each of which consists of a sequence of events which, in turn, a part of the alphabet of event types, grammatical inference can be readily applied to event log data.

The task is to learn a formal representation of the language from which the sample strings in the data originate. These representations are either automata or grammars [6]. Instead of building automata and grammars that decide if strings belong to a language, it is often more useful to build probabilistic versions of these representations [10]. This allows further analyses such as finding the most probable string or predicting how likely certain symbols are given partial strings.

Since it is our goal to apply grammatical inference to the domain of BPM, we focus our discussion on automata. Automata consist of states and transitions between these states. By contrast, grammars consist of production rules that do not directly encode any states. A state-based representation appears most natural as modeling techniques used in BPM follow similar principles. Most importantly, explicit state modeling is one of the reasons for the popularity of Petri nets [11].

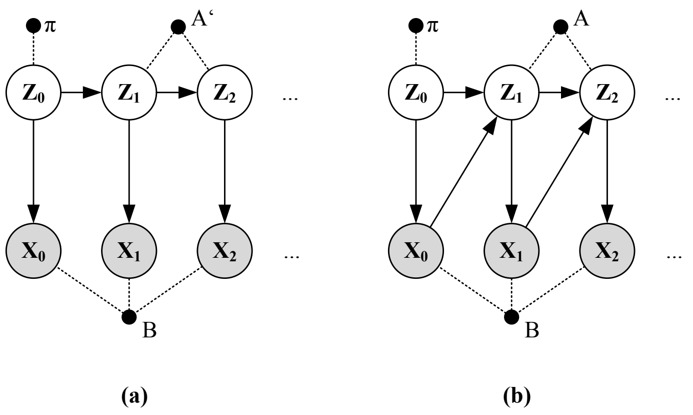


Fig. 1. Directed graphical models of an HMM (a) and a PFA (b). Circles represent random variables and are shaded grey if observations will be available. Directed arcs encode dependencies of random variables among each other. Conversely, missing arcs imply conditional independencies. Parameters are represented as solid black dots and are connected via dashed, undirected arcs to the random variables to whose distributions they belong [12]

There are many different variants of automata studied in the literature. Two of them are most popular [7]. One is the Hidden Markov Model (HMM), the other is the

Probabilistic Finite Automaton (PFA). HMMs [13] are generative models of string distributions. They use an unobserved state space to explain how the strings have been generated. To that end, discrete time steps are defined and the model's state evolves in time. At each step, the HMM is in one state and emits a symbol. The probability of emitting a symbol depends only on the current state. Then, the HMM transitions to another state which it is in in the next time step. The transition probability also depends only on the current step's state. Fig. 1 (a) illustrates the HMM.

Reflecting the HMM's model structure against the backdrop of the BPM domain unveils a problem. The process is driven mainly by the HMM's states and the transitions between them. The symbols however do not influence these transitions. In a typical business process though, we would expect the next state to depend on the event we have just observed. Consider the example of Petri nets, in which the state after firing a transition clearly depends on which transition is chosen. As the HMM assumes independence, we discard it is a probabilistic model for business processes.

This motivates considering the PFA [14]. We illustrated another graphical model in Fig. 1 (b). It is similar to that of the HMM, but with an additional arc from each symbol to the following state. Effectively, this arc removes the problematic assumption of independence. This model can now serve as the probabilistic equivalent of automata as applied in the BPM domain.

2.2 Learning techniques

Approaches to learn probabilistic automata can be categorized broadly into three classes [7]. Those from the first class start with a very complex structure and iteratively merge states until the structure is simple enough. Approaches from the second class assume a standard structure and optimize parameters such that the likelihood of the data is maximal. The third class of approaches consists of sampling methods that rely not on estimating any model but on averaging over many possible models.

In a recent grammatical inference competition, different state-of-the-art learning approaches have been benchmarked against each other on a large collection of data sets [15]. The winning team [16] has designed a collapsed Gibbs sampler to estimate string probabilities, i.e., they applied a method from the third class. Second best scored a team [17] that used maximum likelihood (ML) estimation, i.e., the standard technique from the second class. Methods from the first class have also been applied, yet they scored worse than the others.

These results may indicate that sampling methods are the most promising learning techniques. However, they have an important drawback. While approaches from other categories deliver explicit representations of the analyzed processes along with interpretable parameters, the Gibbs sampling techniques draws randomly many such representations and averages over them when predicting. As a result, there is no way to compactly visualize the predictive model anymore. However, one of our design goals is to avoid creating a black box learning method. Hence, we design our approach not based on sampling techniques but instead based on parameter optimization, i.e., a technique from the second class.

3 PFA learning with event logs

3.1 Adapting the PFA model

In this section, we define two modifications of the PFA model of section 2.1. The first addresses the problem of overfitting to small data sets. It is well-known that an ML estimator is at danger of delivering results that fit well to training data but perform badly on new data [18]. For process data, overfitting can be a severe problem. Process miners consider incompleteness of data as one of the major challenges their discipline has to tackle [19]. Consequently, we have to take precautions.

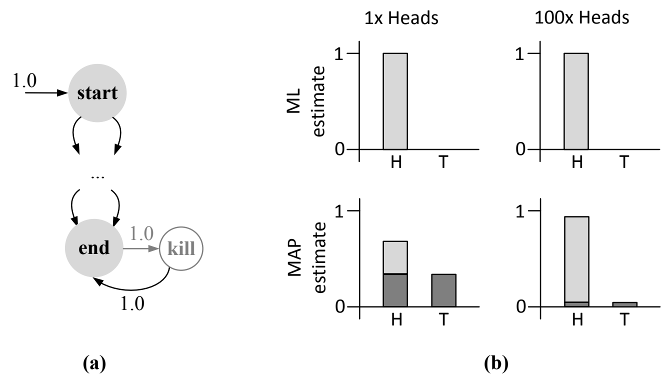
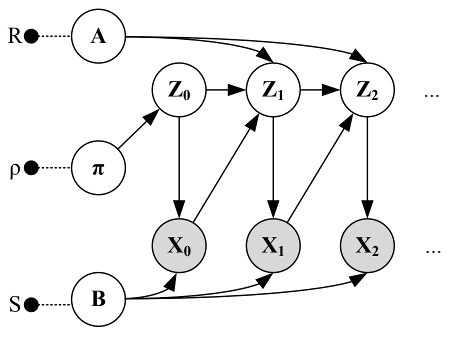


Fig. 2. Illustration of the modifications to the PFA model. (a) illustrates the predefined start and end states. (b) illustrates the effect of Bayesian regularization.

For our probabilistic model, we address this problem with Bayesian regularization [20]. This technique is best illustrated with the simple example of tossing a two-sided coin (cf. Fig. 2 (b) for the following explanation). Consider two data sets. In the first, heads was observed once, in the second, heads was observed 100 times. If the goal is to estimate p, the probability of heads, then for both data sets, the ML estimate will be $p = 1$. In the first scenario, we overfit to the small data set. This can be avoided by smoothing the estimates with artificial observations. In the example of Fig. 2 (b), we can add one artificial observation of both heads and tails, which would deliver an estimate of $p = 2/3$ in the first scenario and $p = 101/102$ in the second. This is achieved by interpreting p not as a parameter but also as a random variable with a distribution. When using the beta distribution in this example, the beta distribution's parameters can be used to specify the artificial observations [21]. Estimating p in such a setting is called Maximum a posteriori (MAP) estimation.

We can move to MAP estimation in our PFA model by treating the parameters as random variables and by defining their distributions to be Dirichlet distributions, which are conjugate priors for the discrete distributions used to define the other variables [21]. The parameters of the Dirichlet distributions can be interpreted as the artificial observations we want to add when estimating parameters of discrete distributions. See Fig. 3. (a) for the graphical model corresponding to the regularized PFA and Fig. 3. (b) for a formal definition of the model.



$$P(Z_0) \sim Discrete(\pi_0, \dots, \pi_K)$$

$$P(X_t | Z_t = k) \sim Discrete(b_{k0}, \dots, b_{kE}),$$
$$\forall k \in \{1, \dots, K\}$$

$$P(Z_t | Z_{t-1} = k, X_{t-1} = e) \sim Discrete(a_{ke0}, \dots, a_{keK}),$$
$$\forall k \in \{1, \dots, K\}, e \in \{1, \dots, E\}$$

$$P(\pi_1, \dots, \pi_K) \sim Dirichlet(\rho_1, \dots, \rho_K)$$

$$P(b_{k1}, \dots, b_{kE}) \sim Dirichlet(s_{k1}, \dots, s_{kE}),$$
$$\forall k \in \{1, \dots, K\}$$

$$P(a_{ke1}, \dots, a_{keK}) \sim Dirichlet(r_{ke1}, \dots, r_{keK}),$$
$$\forall k \in \{1, \dots, K\}, e \in \{1, \dots, E\}$$

(a) **(b)**

Fig. 3. (a) shows the graphical model of the modified PFA. (b) shows the definition of the model's discrete and Dirichlet distributions.

The second modification is motivated by the notion of workflows nets, a special class of Petri nets well known in the BPM domain [11]. Workflow nets have designated start and end places (the *source* and *sink*) since well-designed process models should clearly show where the process starts and ends. We impose the same constraints on the PFA model.

To implement these constraints, we define a special starting state by keeping the vector π fixed at a value that allows the process to start only in that state. In the same way, a special ending state is defined in which only a designated termination event "*kill*" can be emitted. Also, the process is forced to stay in that state. *Kill* shall be an event that can only be emitted in the ending state. All other states have zero probability of emitting it. When processing he event log, the *Kill* must be appended to the end of each process instance. Fig. 2 (a) illustrates the structural assumptions described above. They can all be enforced by keeping the corresponding parameters fixed at suitable values while executing the learning algorithm.

3.2 Learning algorithm

MAP estimation aims at maximizing the posterior probability $P(\theta|X)$. θ denotes all parameters that must be estimated. This optimization can be implemented by optimizing the following expression: $\theta^* = \arg\max_{\theta}[P(X|\theta)P(\theta)]$ [22]. Direct optimization is computationally intractable in presence of unobserved variables such as our state variables. A solution is to apply the expectation maximization (EM) procedure [23]. Starting with (random) initial parameters, it iterates between computing the expectation of the hidden state variables Z with respect to current parameters θ^{old} (i.e., $\mathbb{E}_{Z|X,\theta^{old}} \ln P(X, Z|\theta)$) and deriving updated parameters θ^{new} by optimizing them with respect to this expectation over states (i.e., $\theta^{new} = \arg\max_{\theta}\left[\mathbb{E}_{Z|X,\theta^{old}} \ln P(X, Z|\theta) + \ln P(\theta)\right]$). The algorithm stops once a local optimum is reached [24]. This optimum depends on the initial parameters, which is why it is advisable to run EM multiple times with different initial values to avoid bad local optima [25].

To apply EM to our probabilistic model, we must implement the four steps *initial parameter definition*, *E-step*, *M-step*, and the *check for convergence*. Our implementation generates initial parameters randomly, with the exception that we enforce a starting and ending state as described in section 3.1. The corresponding parameters are fixed at 0 and 1 respectively. Convergence is also determined in the standard way, i.e., by defining a threshold and stopping the algorithm if the likelihood of the data does not improve enough after updating parameters.

To implement the E-step and M-step, we start with $\theta^{new} = \arg\max_{\theta}\left[\mathbb{E}_{Z|X,\theta^{old}} \ln P(X,Z|\theta) + \ln P(\theta)\right]$ and use Lagrangian multipliers to derive updating equations. These updating equations can be found in equations 1-3. They all consist of a *data*-term, which is the contribution of the event log data, and a *prior*-term, which is the contribution of the corresponding Dirichlet distribution. In each M-step, these updating equations are used to compute updated parameter values.

$$\pi_k = \frac{data_k + prior_k}{\sum_{j=1}^{K}(data_k + prior_k)} \quad (1)$$

$$b_{ke} = \frac{data_{ke} + prior_{ke}}{\sum_{e=1}^{E}(data_{ke} + prior_{ke})} \quad (2)$$

$$a_{kej} = \frac{data_{kej} + prior_{kej}}{\sum_{j=1}^{K}\left(data_{kej} + prior_{kej}\right)} \quad (3)$$

In each E-step, the *data*-terms are computed (the *prior*-terms are fixed and require no computations). All terms are shown in equations 4-9. Within these equations, $T_e^{(c)}$ denotes the set of points in time at which an event of type e is observed in instance $x^{(c)}$. $T^{(c)}$ denotes all points in time regardless of the event types. All *data*-terms consist of sums over marginal distributions over the state variables. For our probabilistic model, they can be computed efficiently with a standard procedure called belief propagation [26], which we have implemented in our E-step.

$$data_k = \sum_{c=1}^{C} P\left(Z_0^{(c)} = k \middle| X^{(c)}, \theta^{old}\right) \quad (4)$$

$$prior_k = \rho_k - 1 \quad (5)$$

$$data_{ke} = \sum_{c=1}^{C} \sum_{t \in T_e^{(c)}} P\left(Z_t^{(c)} = k \middle| X^{(c)}, \theta^{old}\right) \quad (6)$$

$$prior_{ke} = s_{ke} - 1 \quad (7)$$

$$data_{kej} = \sum_{c=1}^{C} \sum_{t \in T_e^{(c)}, t \neq 0} P\left(Z_{t-1}^{(c)} = k, Z_t^{(c)} = j \middle| X^{(c)}, \theta^{old}\right) \quad (8)$$

$$prior_{kej} = r_{kej} - 1 \quad (9)$$

3.3 Model selection

The learning algorithm described in the previous section must be provided with some input parameters. To define the dimensions of the probabilistic model, the number of event types E and the number of states K must be given. The former is known, but the latter must be chosen arbitrarily. Hence, we use grid search to determine appropriate values for K. The same is necessary for ρ, S, and R, the parameters of the Dirichlet distributions. As we do not want to bias the model towards certain decisions, we use symmetric Dirichlet priors. However, the strength of these parameters must still be chosen arbitrarily, which again requires grid search.

In grid search, a range of possible values is defined for all parameters not directly optimized by EM. For each combination of values, EM is applied. The trained models from all these runs are subsequently compared to choose one that is best. Different criteria can be applied in this comparison. One solution is assigning a score based on likelihood penalized by the number of parameters, which favors small models. The Akaike Information Criterion (AIC) [27] is a popular criterion of this kind. One disadvantage of the AIC applied to probabilistic models of processes is that we would expect the models to be sparse, i.e., we expect that most transitions have very low probability. Hence, we also consider a less drastic criterion which we call the Heuristic Information Criterion (HIC). It penalizes the likelihood by the number of parameters exceeding a given threshold.

A different solution is to split the event log into a training set used for learning and a validation set used to score generalization performance. According to this criterion, the model performing best on the validation set is chosen.

4 Model structure visualization and analysis

Since we carefully designed the probabilistic model to be interpretable, it is possible to derive meaningful visualizations from a learned model. Using the parameters, we could directly draw an automaton with K states and EK^2 transitions, i.e., a full graph. For all but the smallest examples though, the visualization would not be useful as it would be too complex. However, we can prune the automaton such that only sufficiently probable transitions are kept. For each transition, we delete it if its probability $b_{ke}a_{kej}$ is smaller than a given threshold. As there are KE transitions going out of each state, each transition would have a probability of $1/(KE)$ if all were equally probable. If they are not, then the probability mass will focus at the probable transitions, while probabilities of the improbable transitions will fall below this value. Hence, we define the pruning ratio $prune$ relative to $1/(KE)$. Thresholds are defined as $1/(prune \cdot KE)$, such that a pruning ratio of $prune = 1.5$ means throwing away all transitions being 1.5 times as unlikely as if all transitions had probability $1/(KE)$.

As we effectively construct an automaton from an event log with this procedure, our technique has delivered not only a prediction approach but can also be used as a process discovery algorithm. To this end, we combine it with a technique that was used in the region miner algorithm [9]. In this algorithm, an automaton is constructed and then transformed into a Petri net using a tool called Petrify [28]. Hence, we can also apply

Petrify to transform our pruned automata to Petri nets. We have implemented the entire approach as a process mining plugin for ProM[1].

While discovering processes with our approach might have merits in itself, our primary motivation is to provide non-technical audiences with a visualization of a predictive model. If a stakeholder must be convinced that a certain predictive model should be implemented in his organization, the visualization approach could be used to build trust. Stakeholders can see in form of familiar process modeling notations what the most probable paths through the predictive model are.

5 Evaluation

Experimental setup

To evaluate our approach as a predictive modeling technique, we apply it to a real-life event log from the business process intelligence challenge in 2012 [29]. It originates from a Dutch financial institute and contains about 262,000 events in 13,084 process instances. There are three subprocesses (called P_W, P_A, and P_O) and each event is uniquely assigned to one subprocess. Apart from the complete event log, we generated three additional event logs by splitting the complete log into these subprocesses. Each of the four event logs is split into a training set (50%), a validation set (25%) and a test set (25%). The training set is used to run EM, the validation set is used during model selection, and the test set is used to assess final performance.

The EM algorithm was configured in the following way. Convergence was achieved when the log-likelihood increased less than 0.001 after the M-step. As a grid for K, we used $[2,4,8,10,12,14,16,18,20,30,40]$. The grid for the Dirichlet distributions' strength was defined relative to the sizes of the event logs. The overall number of artificial observations, distributed symmetrically over all Dirichlet distributions, was optimized over the grid $[0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.75]$ and is expressed as a fraction of the total number of events in the logs. To compare different models in grid search, we separately tried all three methods described in section 3.3, with the HIC applied with a threshold of 0.05.

One measure to estimate the quality is H, the cross entropy (as estimated w.r.t. the test set). We also consider a number of different prediction problems that could occur in practice. The most straightforward one is predicting what the next symbol will be. We report the average accuracy of our predictors. Accuracy is the fraction of predictions in which the correct event has been predicted. The other prediction problem is concerned with specific types of events. After selecting an event type, the task is to predict whether or not the next event will be of this type. We measured the sensitivies (true positives) and specitivities (true negatives) for each predictor applied to the problem of deciding this for each event.

Note that our approach is not restricted to predicting the next event. Evaluating the distribution over an event arbitrarily far in the future is also feasible. However, our evaluation focusses on predicting the next event only.

[1] http://goo.gl/FqzJwp

Results

In Table 1, we list the results for all four event logs (P_W, P_A, P_O, and $P\_Complete$). For each, we report performance statistics for the three different model selection criteria (EM_HIC, EM_AIC, and EM_Val). The sensitivities and specificities have been averaged over all symbols to keep the table compact.

A first observation is that accuracies do not rise up close to 1. However, accuracies that high cannot be expected, since there is inherent uncertainty in the decisions made in business processes. Thus, predicting the next event correctly must fail in some cases. Predicting events completely at random would deliver the correct event with probability $1/E$. For the three events logs, accuracies of random prediction would be below 0.10. Hence, our approach is significantly better than random guessing.

As for comparing the model selection criteria, the results demonstrate that using the validation set to assess performance delivered solid results if measured by cross entropy. For this metric, the validation set always delivered the best model, i.e., the one with the smallest cross entropy.

This is changing though if we turn to the actual prediction problems. The HIC performed best on $P\_Complete$ and P_A, yet bad on P_W, for which the AIC worked better. For event log P_O, all three methods deliver nearly the same performance. Hence, our preliminary results indicate that no model selection criterion can generally outperform the others.

Table 1. Accuracy, average sensitivity and average specitivity for different predictive models.

Event log	Predictor	Accuracy	Ø Sensitivity	Ø Specitivity	H
P_W	EM_HIC	0.685	0.558	0.950	12.810
	EM_AIC	0.719	0.578	0.955	11.183
	EM_Val	0.718	0.582	0.955	10.385
P_A	EM_HIC	0.801	0.723	0.980	3.093
	EM_AIC	0.769	0.658	0.977	3.393
	EM_Val	0.796	0.720	0.980	3.014
P_O	EM_HIC	0.809	0.646	0.973	4.588
	EM_AIC	0.811	0.647	0.973	4.513
	EM_Val	0.811	0.647	0.973	4.513
P_Complete	EM_HIC	0.735	0.508	0.988	17.273
	EM_AIC	0.694	0.460	0.986	17.726
	EM_Val	0.700	0.437	0.986	16.078

In Fig. 4, an exemplary Petri net visualization is shown. It has been generated from a predictor for event log P_A, chosen by the HIC. An analyst could double-check this visualization with his domain knowledge. Alternatively, he could apply other process discovery techniques to verify that the predictive model is indeed a good model for the event log. Comparing this Petri net with results found during the challenge, we can see that the visualization of the predictor is almost identical to what has been discovered with other methods [30]. Hence, this visual comparison with other findings delivers further evidence for the appropriateness of the predictive model.

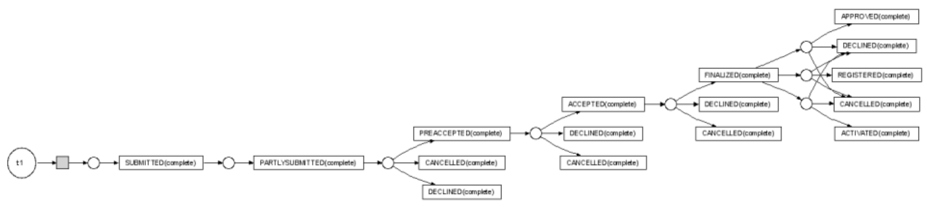


Fig. 4. Petri net visualization of predictor fitted with the HIC criterion, generated with the ProM plugin and a pruning ratio of 1.5.

6 Conclusion and Outlook

In this paper, we presented an approach to apply predictive modeling to event log data. We discussed different representations used in the literature of grammatical inference to model formal languages probabilistically. Against the backdrop of the BPM domain, we identified probabilistic versions of finite automata to be most appropriate. We also identified suitable learning techniques to fit these models to data. Starting with this theoretical background, we modified the probabilistic model to tailor it to the problem at hand. Subsequently, we derived an EM optimization algorithm, defined a grid search procedure to also optimize its inputs, and tested the approach on real-world event logs. Our preliminary results demonstrate the applicability of the approach. With the ProM plugin, in which we combined the predictive approach with ProM's Petri net synthesis capabilities, we provide a readily applicable process discovery tool.

In future research, we plan to conduct more extensive evaluations. In particular, we will systematically compare our model selection criteria to identify which work best under which circumstances. Adding experiments with synthetic data may also allow us to thoroughly investigate the performance of the predictive approach with respect to a known gold standard. For the real-life event log we used in the preliminary evaluation, the "true" probabilities are unknown, so that we cannot quantify the quality of the predictors generated with our approach in absolute terms.

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